SOLUTIONS MATHEMATICS IGCSE P1 V2

Q. No. 1:

i) a) $15 - 5\times(7 - 5) \div 4 + 9 = 21.5$

b) $11 + 88 \div (1 + 1) \times 11 = 495$

ii) This is not possible, which means no solution satisfies all four congruences. Thus, no greatest 5-digit number satisfies the given conditions.

iii) a) The number 10203005 has 8 significant digits.

(b) The number 23.0561 has 6 significant digits.

iv) For 1/2:

Percentage = $(1/2)\times100\% = 50\%$

For 5/2:

Percentage = $(5/2) \times 100\% = 250\%$

For 43%:

Fraction = $(43/100) = 0.43$

Q. No. 2:

i) Given:

- The probability of the bus being early = 0.1
- The probability of the bus being on time = 0.6

Let

P(late) be the probability that the bus is late. We have:

P(early) + P(on time) + P(late) = 1

0.1 + 0.6 + P(late) = 1

P(late) = 1 − 0.1 − 0.6

P(late) = 0.3

ii) If the term-to-term rule for the sequence is "multiply by 8 and then add 1," we can express the sequence as follows:

- Let the first term be a.
- The second term is obtained by multiplying the first term by 8 and then adding 1, which gives

  $8a+1$.

- The third term is obtained by multiplying the second term by 8 and then adding 1, which gives

$8(8a + 1) + 1$.

Therefore, the sequence with three terms is:

1st term: a

2nd term: 8a + 1

3rd term: 8(8a + 1) + 1

Simplifying the third term:

8(8a + 1) + 1 = 64a + 8 + 1 = 64a + 9

So, the sequence with three terms is a, 8a + 1,64a + 9.

iii) To convert 17 gallons into litres using the conversion factor 2 gallons = 9 litres:

Calculate how many sets of 2 gallons are in 17 gallons:

17 gallons ÷ 2 gallons = 8.5

Multiply the number of sets by 9 litres to get the equivalent in litres:

8.5 sets × 9 liters per set = 76.5 liters

So, 17 gallons is equivalent to 76.5 litres.

iv)

Given that the ratio of the base b to the side s is 2:3, we can express s in terms of b as follows:

$$s = \frac{3}{2}b$$

Therefore, the perimeter P of the trapezium is:

$$P = b + b + s + s$$

Substituting s with $\frac{3}{2}b$:

$$P = 2b + 2\left(\frac{3}{2}b\right)$$

Simplifying the expression:

$$P = 2b + 3b$$

$$P = 5b$$

So, the expression for the perimeter of the trapezium in terms of b is $5b$ centimeters.

v) a) Relative frequency of landing on heads = $\frac{31}{50}$.

b) or a fair coin, we would expect the probability of landing on heads to be 0.5 (50% chance). However, in this case, the coin landed on heads 31 times out of 50, which corresponds to a relative frequency of $\frac{31}{50}$ = 0.62 or 62%.

Since the relative frequency of landing on heads is higher than the expected 50% for a fair coin, it suggests that the coin may indeed be biased towards heads.

Q. No. 3:

i) a) cosx = − cos60°.

Using the identity

cos(− θ) = cos(θ), we can rewrite this as

cosx = cos60°.

Since the cosine function is positive in the second and third quadrants, where

180° ≤ x ≤ 360°, the solution is x = 300°.

b) cosx = cos60°.

The solutions to this equation occur when x is either 60° or 300° (since cosine is positive in the first and fourth quadrants, and 90° ≤ x ≤ 360°).

ii) The formula to find the interior angle of a regular polygon is:

$$\text{Interior angle} = \frac{(n-2)\times 180^\circ}{n}$$

where n is the number of sides of the polygon.

Given that the interior angle of the regular polygon is 156°, we can set up the equation:

$$156 = \frac{(n-2)\times 180}{n}$$

Solving for n:

156n = 180(n − 2)

156n = 180n − 360

360 = 24n

N = 15

So, the regular polygon has 15 sides.

iii) a)

$$\text{Speed} = \frac{500000}{2.333}\ \mathrm{m/h} \approx 214285.71\ \mathrm{m/h}$$

$$\text{Speed} \approx \frac{214285.71}{3600}\ \mathrm{m/s} \approx 59.52\ \mathrm{m/s}$$

b)

$$\text{Speed} = \frac{500}{2.333}\ \text{km/h} \approx 214.29\ \text{km/h}$$

$$\text{Speed} \approx \frac{214.29}{60}\ \text{km/min} \approx 3.57\ \text{km/min}$$

Q. No. 4:

i) a)

	5	6	7	8
H	H5	H6	H7	H8
T	T5	T6	T7	T8

b) The probability of getting a tail with an even number is the number of favourable outcomes (getting a tail with a 6 or 8) divided by the total number of outcomes.

Number of favorable outcomes = 2 (T6 and T8)

Total number of outcomes = 4 (T5, T6, T7, T8)

Therefore, the probability is $\frac{2}{4} = \frac{1}{2}$ when expressed in its simplest form.

ii) The perimeter of an equilateral triangle is three times the length of one side, and the perimeter of a square is four times the length of one side.

Given that the perimeter of each shape is 36 cm, we can set up the following equations:

For the equilateral triangle:

$$3a = 36$$

$$a = \frac{36}{3}$$

$$a = 12$$

For the square:

$$4(a+b) = 36$$

$$4(12+b) = 36$$

$$48 + 4b = 36$$

$$4b = 36 - 48$$

$$4b = -12$$

$$b = \frac{-12}{4}$$

$$b = -3$$

iii) To show that triangles ABC and EDC are similar, we need to show that their corresponding angles are equal.

In triangle ABC, the angle B is common to both triangles.

In triangle EDC, angle D is vertically opposite to angle B, so the angle D=∠B.

Angle C in triangle ABC is equal to angle E in triangle EDC because they are corresponding angles between parallel lines AE and BD.

Therefore, we have:

$\angle B = \angle D$

$\angle C = \angle E$

Since all corresponding angles are equal, by the angle-angle (AA) similarity criterion, triangles

ABC and EDC are similar.

Q. No. 5:

i) Subtracting the second equation from the first to eliminate y:

$(4x + 3y) - (2x + 3y) = 5 - 1$

$4x + 3y - 2x - 3y = 4$

$2x = 4$

$X = 2$

Now, substitute $x=2$ into one of the original equations to find y.

Using the second equation:

$2(2) + 3y = 1$

$4 + 3y = 1$

$3y = 1 - 4$

$3y = -3$

$Y = -1$

ii)

- Calculate the volume of each wood panel:

  Volume = length × width × height

  Volume = 2.4 m × 1.2 m × 0.018 m (convert 1.8 cm to meters)

  Volume = 0.0648 m^3

- Calculate the total weight of each wood panel:

  Weight = Density × Volume

  Weight = 750 kg/m^3 × 0.0648 m^3

  Weight = 48.6 kg

- Calculate the maximum number of wood panels the truck can carry:

  Maximum weight capacity of the truck = 15 tonnes = 15,000 kg

Number of panels = Maximum weight capacity of the truck / Weight of each panel

Number of panels = 15,000 kg / 48.6 kg ≈ 308.64

Since the number of panels must be a whole number, the maximum number of wood panels that the truck can carry is 308 panels.

iii) The formula y = xk, where k is a constant of proportionality.

Given that y is 40 when x is 9, we can substitute these values into the formula to find k:

$$40 = \frac{k}{\sqrt{9}}$$

$$40 = \frac{k}{3}$$

K = 40 × 3

K = 120

So, the formula linking x and y is :

$$y = \frac{120}{\sqrt{x}}$$

Q. No. 6:

i) a) The radius of the larger circle is the sum of the radius of the pond and the width of the path, which is 6+1=7 meters.

The area of the larger circle is $\pi \times (7^2)$.

The radius of the smaller circle is the radius of the pond, which is 6 meters.

The area of the smaller circle is $\pi \times (6^2)$.

Therefore, the area of the path is the difference between the area of the larger circle and the area of the smaller circle:

$$\text{Area of path} = \pi \times (7)^2 - \pi \times (6)^2$$

$$\text{Area of path} = \pi \times 49 - \pi \times 36$$

$$\text{Area of path} = \pi \times (49 - 36)$$

$$\text{Area of path} = \pi \times 13$$

b) The total area of the path is 13π square meters.

The area covered by one pack of astroturf is 7 square meters.

Therefore, the number of packs required is:

$$\text{Number of packs} = \frac{13\pi}{7}$$

$$\text{Number of packs} \approx \frac{13\times3.14159}{7}$$

$$\text{Number of packs} \approx \frac{40.84067}{7}$$

$$\text{Number of packs} \approx 5.83438$$

ii) The volume of a prism is given by the formula V = Area of base × Height, where the base is an equilateral triangle in this case.

Since the prism has a volume of 100 and a length (height) of 8, we can write:

100 = Area of base × 8

Area of base = 12.5

The area of an equilateral triangle is given by the formula:

$$\frac{\sqrt{3}}{4} \times \text{side}^2.$$

$$12.5 = \frac{\sqrt{3}}{4} \times \text{side}^2$$

$$\text{side}^2 = \frac{12.5\times4}{\sqrt{3}}$$

$$\text{side}^2 = \frac{50}{\sqrt{3}}$$

$$\text{side} = \sqrt{\frac{50}{\sqrt{3}}}$$

$$\text{side} = \sqrt{\frac{50\times\sqrt{3}}{3}}$$

$$\text{side} = \sqrt{\frac{50\times\sqrt{3}\times\sqrt{3}}{3\times3}}$$

$$\text{side} = \sqrt{\frac{150}{9}}$$

$$\text{side} = \sqrt{\frac{50}{3}}$$

Q. No. 7:

i) a) Two heads and a total of 12 on the dice:

Number of ways to get two heads with two coins: 1 (HH)

Number of ways to get a total of 12 on two dice: 1 (6+6)

Total number of outcomes for two coins and two dice:

$$2 \times 2 \times 6 \times 6 = 144$$

Number of favourable outcomes: 1 (HH, 6+6)

Probability: $\frac{1}{144}$

b) A head, a tail, and a total of 9 on the dice:

Number of ways to get a head and a tail with two coins: 2 (HT, TH)

Number of ways to get a total of 9 on two dice: 4 (3+6, 4+5, 5+4, 6+3)

Total number of outcomes for two coins and two dice:

$$2 \times 2 \times 6 \times 6 = 144$$

Number of favourable outcomes: 2 (HT, TH) $\times$ 4 (3+6, 4+5, 5+4, 6+3) = 8

Probability: $\frac{1}{18}$

c) Two tails and a total of 3 on the dice:

Number of ways to get two tails with two coins: 1 (TT)

Number of ways to get a total of 3 on two dice: 2 (1+2, 2+1)

Total number of outcomes for two coins and two dice:

$$2 \times 2 \times 6 \times 6 = 144$$

Number of favourable outcomes: 1 (TT) * 2 (1+2, 2+1) = 2

Probability: $\frac{1}{72}$

The most likely outcome is the one with the highest probability, which in this case would be obtaining a head, a tail, and a total of 9 on the dice.

ii) a) $(81 + 80)(81 - 80)$

$=161 \times 1$

$=161$

b) $6m + 4n - 9km - 6kn$

$= 4m - 9km + 4n - 6kn$

$= m(4 - 9k) + n(4 - 6k)$

$= (m + n)(4 - 9k)(4 - 6k)$

Q. No. 8:

i) Surface area of the spherical part of the ball: $4\pi \times (20.5)^2$ mm².

Surface area of one dimple: $2\pi \times (2)^2$ mm².

$$\text{Total surface area} = 4\pi \times (20.5)^2 + 150 \times 2\pi \times (2)^2$$

Calculating the total surface area:

$$\text{Total surface area} = 4\pi \times (20.5)^2 + 150 \times 2\pi \times (2)^2$$

$$= 4\pi \times 420.25 + 150 \times 2\pi \times 4$$

$$= 1681\pi + 1200\pi$$

$$= 2881\pi$$

Using the value of $\pi \approx 3.14159$:

$$\text{Total surface area} \approx 2881 \times 3.14159$$

$$\approx 9058.88 \text{ mm}^2$$

ii) a) x = $180° - 45° - 70° = 65°$

b) $x^2 = 180° - 110°$

x = 35°

c) y = $180° - 62° - 51° = 67°$

x = $180° - 67° = 113°$

d) x = $180° - 120° = 60°$

y = $180° - 60° - 70° = 50°$

Q. No. 9:

i) a)

The percentage of shaded boxes is calculated as:

$$\frac{\text{Number of shaded boxes}}{\text{Total number of boxes}} \times 100\%$$

Substituting the numbers:

$$\frac{2}{7} \times 100\%$$

b) The percentage of shaded boxes is 28.57%.

ii) Given equations:

2x + 5y = 24

4x + 3y = 20

We can multiply the first equation by 2 and the second equation by 5 to make the coefficients of y the same and then eliminate Y:

4x + 10y = 48

20x + 15y = 100

Now, subtract the second equation from the first to eliminate y:

(4x + 10y) − (20x + 15y) = 48 − 100

− 16x − 5y = − 52

Now, solve for x:

−16x − 5y = −52

−16x = −52 + 5y

$x = \frac{52-5y}{16}$

Substitute x back into the first equation to solve for y:

$2(\frac{52-5y}{16}) + 5y = 24$

$\frac{104-10y}{16} + 5y = 24$

$104 - 10y + 80y = 384$

$70y = 280$

$y = \frac{280}{70}$

$y = 4$

Now, substitute $y = 4$ back into $x = \frac{52-5y}{16}$ to solve for x:

$x = \frac{52-5(4)}{16}$

$x = \frac{52-20}{16}$

$x = \frac{32}{16}$

$x = 2$