SOLUTIONS MATHEMATICS IGCSE P1 V3

Q. No. 1:

i) a) 7 + (− 11) + 5 = − 4 + 5 = 1

b) − 8 + 10 − (− 17) + 10

= 2 + 17 + 10 = 19 + 10 = 29

ii) a)

$$(k + m + n)/(k^2 + m^2 + n^2)$$

$$(-3 + 1 - 4)/((-3)^2 + 1^2 + (-4)^2)$$

$$(-6)/(9 + 1 + 16)$$

$$(-6)/(26)$$

Now, we simplify the fraction:

$$-6/26 = -3/13$$

b)

$$k^2m^2(m - n)$$

$$(-3)^2 \times 1^2(1 - (-4))$$

$$9 \times 1 \times 5$$

$$45$$

c)

$$m\sqrt{(k - n)}$$

$$1\sqrt{(-3 - (-4))}$$

$$1\sqrt{(1)}$$

$$1$$

Q. No. 2:

i) a)

Left side: $x^2 + 5x + 4$

Right side: $x^2 - x - 42$

Setting them equal: $x^2 + 5x + 4 = x^2 - x - 42$

Subtracting x^2: $6x + 4 = -x - 42$

Adding x: $7x + 4 = -42$

Subtracting 4: $7x = -46$

Dividing by 7: $x = -46/7$

So, the solution is $x = -46/7$.

b)

$\frac{3(2x+1)}{3\times8} - \frac{8(x-1)}{8\times3} = \frac{5}{24}$

$\frac{6x+3}{24} - \frac{8x-8}{24} = \frac{5}{24}$

Combining the fractions:

$\frac{6x+3-8x+8}{24} = \frac{5}{24}$

$\frac{-2x+11}{24} = \frac{5}{24}$

Since the denominators are the same, we can equate the numerators:

$-2x + 11 = 5$

$-2x = -6$

$x = 3$

c)

Let the two numbers be x and $11x$, since they are in the ratio 1:11.

According to the problem, their sum is 15:

$x + 11x = 15$

$12x = 15$

$x = \frac{15}{12}$

$x = \frac{5}{4}$

So, the two numbers are $\frac{5}{4}$ and $\frac{55}{4}$.

iii) a) From equation 2, we have

x = 13 − 5w.

Substitute x in equation 1:

2w + 39 − 15w − 13=0

− 13w + 26 = 0

−13w = − 26

W = 2

Substitute w = 2 back into X = 13 − 5w:

X = 13 − 5(2)

X = 13 − 10

X = 3

So, the solution is w = 2 and x = 3.

b)

$$ax - ay - bs + by = a(x - y) - b(s - y)$$

Q. No. 3:

i) a) 458 is much larger than the next term we would get from this pattern. Therefore, 458 is not a term in the given sequence.

b)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this equation, $a = 2, b = -3$, and $c = -1$. Plugging these values into the formula:

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 2 \times (-1)}}{2 \times 2}$$

$$x = \frac{3 \pm \sqrt{9+8}}{4}$$

$$x = \frac{3 \pm \sqrt{17}}{4}$$

So, the solutions are:

$$x = \frac{3+\sqrt{17}}{4} \approx 1.56$$

$$x = \frac{3-\sqrt{17}}{4} \approx -0.31$$

iii)

$$y - y_1 = m(x - x_1)$$

So, the equations of the lines are:

a) $y = 3x - 2$

b) $y = 7x - 9$

c) $y = 2x - 19$

d) $y = \frac{1}{2}x + 1$

e) $y = -\frac{1}{2}x + 3$

Q. No. 4:

i) a)

$$\text{Interior angle} = \frac{(n-2)\times 180^\circ}{n}$$

where n is the number of sides of the polygon. Since the interior angle is given as 156°, we can set up the equation:

$$156 = \frac{(n-2)\times 180}{n}$$

Simplifying:

$$156n = 180n - 360$$

$$24n = 360$$

$$n = \frac{360}{24}$$

$$n = 15$$

b)

$$150 = \frac{(n-2)\times 180}{n}$$

Simplifying:

$$150n = 180n - 360$$

$$30n = 360$$

$$n = \frac{360}{30}$$

$$n = 12$$

ii) a = 55°

c = $180° - 123° = 57°$

b = c = 57°

d = $180° - a - c = 68°$

Q. No. 5:

i) The second triangle with sides 7.2 cm, 9.6 cm, and 12 cm is a right-angled triangle. This is because $12^2 = 7.2^2 + 9.6^2$ satisfies the Pythagorean theorem, making it a right-angled triangle.

ii)

$$\text{Area} = \frac{24 \times 24}{2} = \frac{576}{2} = 288 \text{ cm}^2$$

iii)

$$\text{Area} = \frac{5}{2} \times \text{side} \times \text{apothem}$$

$$\text{Area} = \frac{5}{2} \times 8 \times 3.0777 \approx 38.472 \text{ cm}^2$$

Q. No. 6:

i) a)

1. **Area:** $A = \pi \times 5^2 = \pi \times 25 = 25\pi$ square cm. (Approximately 78.54 square cm.)
2. **Perimeter:** $P = 2\pi \times 5 = 10\pi$ cm. (Approximately 31.42 cm.)

b)

1. **Radius:** The radius (r) of a circle is half of its diameter. So, $r = \frac{24}{2} = 12$ meters.
2. **Area of a Semicircle:** The area (A) of a semicircle is half the area of a circle with the same radius. So, $A = \frac{1}{2} \times \pi \times r^2$.
   Plugging in the radius, $A = \frac{1}{2} \times \pi \times 12^2 = \frac{1}{2} \times \pi \times 144 = 72\pi$ square meters.
3. **Perimeter of a Semicircle:** The perimeter (P) of a semicircle is the sum of the semicircular arc and the diameter. The semicircular arc is half the circumference of a full circle. So, $P = \frac{1}{2} \times 2\pi r + 2r$.
   Plugging in the radius, $P = \frac{1}{2} \times 2\pi \times 12 + 2 \times 12 = \pi \times 24 + 24 = 24(\pi + 1)$ meters.

c)

1. **Full Circle:**
   - **Radius:** $r = 1.3 \text{ cm}$
   - **Area of Circle:** $A_{\text{circle}} = \pi r^2$
     $A_{\text{circle}} = \pi \times (1.3)^2 \approx 5.309 \text{ cm}^2$
   - **Perimeter of Circle:** $P_{\text{circle}} = 2\pi r$
     $P_{\text{circle}} = 2 \times \pi \times 1.3 \approx 8.168 \text{ cm}$
2. **3/4 Circle:**
   - **Area of 3/4 Circle:** $A_{3/4} = \frac{3}{4} \times A_{\text{circle}}$
     $A_{3/4} = \frac{3}{4} \times 5.309 \approx 3.982 \text{ cm}^2$
   - **Perimeter of 3/4 Circle:** $P_{3/4} = \frac{3}{4} \times P_{\text{circle}}$
     $P_{3/4} = \frac{3}{4} \times 8.168 \approx 6.126 \text{ cm}$

ii) Yes, Lakmini's friend was correct. Taking off 6 m from Lakmini's measured circumference brings the measurement closer to the actual circumference of the Earth.

iii)

Volume of the cylinder = $\pi r^2 h$

Volume of the cube = s^3

Given: radius $r = 4$ cm, height $h = 8$ cm

Volume of the cylinder = $\pi \times 4^2 \times 8 = 128\pi$ cm³

Since the volume of the cylinder is equal to the volume of the cube:

$s^3 = 128\pi$

Taking the cube root of both sides:

$s = \sqrt[3]{128\pi} \approx 6.85$ cm

So, the side length of the cube is approximately 6.85 cm.

iv) Total volume = $V_{cylinder} + V_{hemisphere} + V_{cone}$

$= 90\pi + 18\pi + 24\pi$

$= 132\pi cm^3$.

Q. No. 7:

i) the total number of outcomes when throwing two dice and two coins is 36 × 4 = 144.

a) The probability of this outcome is $\frac{1 \times 1}{144} = \frac{1}{144}$.

b) The probability of this outcome is $\frac{2 \times 2}{144} = \frac{4}{144} = \frac{1}{36}$.

c) Probability of two tails and a total of 3 on the dice $\frac{1}{72}$.

The most likely outcome is obtaining a head, a tail, and a total of 9 on the dice, with a probability of $\frac{1}{36}$.

ii) a) The probability of drawing two green balls is $\frac{3}{8} \times \frac{3}{8} = \frac{9}{64}$.

b) the probability of the first ball being red and the second being green is $\frac{5}{8} \times \frac{3}{8} = \frac{15}{64}$.

iii) a) The median age is 16 years.

b) The median height is 170 cm.

c) The median weight is 48 kg.

Q. No. 8:

i) a)

Mean = $\frac{1\times4+2\times11+3\times7+4\times x}{4+11+7+x}$

Given that the mean is $\frac{21}{3}$, we have:

$\frac{21}{3} = \frac{4+22+21+4x}{22+x}$

Multiplying both sides by 22 + x to get rid of the denominator:

$7(22+x) = 4+22+21+4x$

$154+7x = 47+4x$

$3x = -107$

$x = -\frac{107}{3}$

b) The mode is the value that appears most frequently. Since the mode is 2, the largest possible value of x occurs when there are 11 cars with 2 occupants, and no other value for x satisfies this condition. So, the largest possible value of x is 11.

c) The mode is the value that appears most frequently. Since the mode is 2, the largest possible value of x occurs when there are 11 cars with 2 occupants, and no other value for x satisfies this condition. So, the largest possible value of x is 11.

ii) Probability of drawing a red ball from the initial bag and then adding a red ball $\frac{1}{2}\times\frac{4}{7}=\frac{2}{7}$.

Probability of drawing a blue ball from the initial bag and then adding a red ball $\frac{1}{2}\times\frac{3}{7}=\frac{3}{14}$.

The total probability of drawing a red ball after the process is the sum of these probabilities:

$$\frac{2}{7}+\frac{3}{14}=\frac{4}{14}+\frac{3}{14}=\frac{7}{14}=\frac{1}{2}$$

iii) Population: A population is the entire group of individuals, events, or things that we are interested in studying or describing.

Sample: A sample is a subset of the population that is selected to represent the population.