SOLUTION MATHEMATICS IGCSE P1 V4

Q. No. 1:

i) Multiplying the numerator and denominator of five-eighths by the same number (in this case, 3) to get an equivalent fraction with a denominator of 24:

$$\frac{5}{8} \times \frac{3}{3} = \frac{15}{24}$$

ii)

a) $\frac{2}{5}$ of 25g

$\frac{2}{5} \times 25 = \frac{2 \times 25}{5} = \frac{50}{5} = 10$

So, $\frac{2}{5}$ of 25g is 10g.

b) $\frac{3}{7}$ of 30m

$\frac{3}{7} \times 30 = \frac{3 \times 30}{7} = \frac{90}{7}$

c) $\frac{4}{9}$ of 45 gallons

$\frac{4}{9} \times 45 = \frac{4 \times 45}{9} = \frac{180}{9} = 20$

d) $\frac{3}{4}$ of 420 miles

$\frac{3}{4} \times 420 = \frac{3 \times 420}{4} = \frac{1260}{4} = 315$

iii)

$\frac{1}{x^2}$

This can also be written as x^{-2}, using negative exponents.

Q. No. 2:

i)

a) 84, 56

$$84 = 56 \times 1 + 28$$
$$56 = 28 \times 2 + 0$$

The last non-zero divisor is 28, so the HCF of 84 and 56 is 28.

b) 21, 68, 77

First, find the HCF of the first two numbers, then find the HCF of that result and the third number.

$68 = 21 \times 3 + 5$
$21 = 5 \times 4 + 1$
$5 = 1 \times 5 + 0$

The last non-zero divisor is 1, so the HCF of 21 and 68 is 1.

Now, find the HCF of 1 and 77:

$77 = 1 \times 77 + 0$

The last non-zero divisor is 1, so the HCF of 21, 68, and 77 is 1.

ii)

To simplify $\sqrt{132}$, we first look for perfect square factors of 132:

$$\begin{aligned} 132 &= 2 \times 66 \\ &= 2 \times 2 \times 33 \\ &= 2 \times 2 \times 3 \times 11 \end{aligned}$$

So, $\sqrt{132} = \sqrt{2 \times 2 \times 3 \times 11} = 2 \times \sqrt{33}$.

iii)

$\text{Side length} = \sqrt{361} = 19 \text{ inches}$

Perimeter = 4 × 19 = 76 inches

To convert this to feet, we divide by 12 (since there are 12 inches in a foot):

$\text{Perimeter in feet} = \frac{76}{12} = 6.33 \text{ feet}$

iv) The ratio of blue triangles to all triangles is 1:4.

Q. No. 3:

i) a)

$-12 \times \frac{4}{1} = -48$

b)

$(-2) \times (-2) \times (-2) = -8.$

c) This is a difference of squares, so we can factor it as (x + 9)(x − 9) = 0. Setting each factor to zero gives x + 9 = 0 or x – 9 = 0, so x = – 9 or x = 9.

d)

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Here, $a = 1, b = 6$, and $c = 4$. Plugging these values into the formula:

$x = \frac{-6 \pm \sqrt{6^2-4\times1\times4}}{2\times1}$
$x = \frac{-6 \pm \sqrt{36-16}}{2}$
$x = \frac{-6 \pm \sqrt{20}}{2}$
$x = \frac{-6 \pm 2\sqrt{5}}{2}$
$x = -3 \pm \sqrt{5}$

So, the solutions are $x = -3 + \sqrt{5}$ and $x = -3 - \sqrt{5}$.

e)

First, simplify the division by multiplying by the reciprocal of the divisor:

$\frac{9x^2-4}{3x-2} \times \frac{5x}{9x+6}$

Factor the numerator:

$\frac{(3x-2)(3x+2)}{3x-2} \times \frac{5x}{3(3x+2)}$

Cancel out the common factor in the numerator and denominator:

$\frac{3x+2}{3} \times \frac{5x}{3x+2}$

Now, multiply:

$\frac{5x}{3}$

So, $\frac{9x^2-4}{3x-2} \div \frac{9x+6}{5x} = \frac{5x}{3}$.

ii)

$$W = \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{3}\right)\left(\frac{1}{4}\right)\left(\frac{1}{12}\right)}{\left(\frac{1}{6}\right)\left(\frac{5}{4}\right)}$$

Multiply the numerators and denominators:

$$W = \frac{\frac{1}{24} \times \frac{1}{12}}{\frac{5}{24}}$$

Simplify the numerator:

$$W = \frac{\frac{1}{288}}{\frac{5}{24}}$$

Divide the fractions:

$$W = \frac{\frac{1}{288}}{\frac{5}{24}} = \frac{1}{288} \times \frac{24}{5} = \frac{24}{288} \times \frac{1}{5} = \frac{1}{12} \times \frac{1}{5} = \frac{1}{60}$$

So, $W = \frac{1}{60}$.

Q. No. 4:

i) a)

$$(2a - 3b)^2 = (2a)^2 - 2(2a)(3b) + (-3b)^2$$

$$= 4a^2 - 12ab + 9b^2$$

So, $(2a - 3b)^2 = 4a^2 - 12ab + 9b^2$.

b)

$$100x = -6 + 5$$

$$100x = -1$$

Then, divide by 100 to solve for x:

$$x = \frac{-1}{100}$$

So, $x = -\frac{1}{100}$.

ii) David's mass is 5 kg less than John's: D = J – 5

John's mass is 8 kg lighter than Paul's: J = P – 8

Given that their total mass is 197 kg:

P + (P – 8) + (P – 8 – 5) = 197

P + P – 8 + P – 13 = 197

3P – 21 = 197

3P = 218

$$P = \frac{218}{3}$$

So, Paul's mass is P = 72.67 kg approximately.

Now, we can find John's and David's masses:

John's mass:

J = P – 8 = 72.67 – 8 = 64.67 kg

David's mass:

D = J–5 = 64.67−5 = 59.67 kg

iii) 2T + 3D = 1750 (the cost of two televisions and three DVD players is $1750)

4T + D = 1250 (the cost of four televisions and one DVD player is $1250)

We can solve these equations simultaneously to find the values of From equation 2, we can express D in terms of D = 1250 − 4T

Substitute this into equation 1:2T + 3(1250 − 4T) = 1750

Simplify: 2T + 3750 − 12T = 1750

−10T = −2000

T = 200

Now that we have the cost of one television, we can find the cost of one DVD player using equation 2:

D = 1250 – 4 × 200

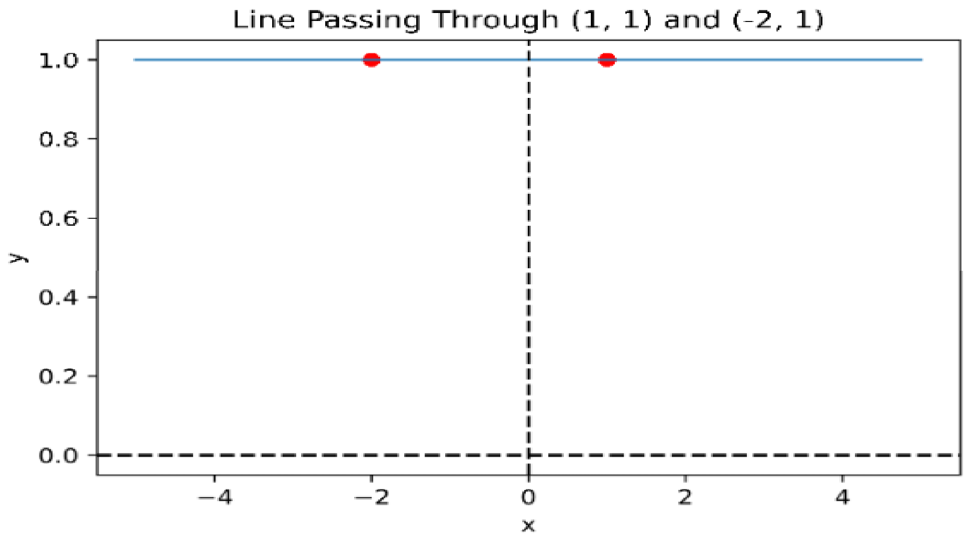
D = 1250 − 800

D = 450

So, the cost of one television is $200 and the cost of one DVD player is $450.

Q. No. 5:

i)



ii) a) Decagon

b) Heptagon

iii)

A 5CM C

iv) a)

$x^2 = 5^2 + 12^2$

$x^2 = 25 + 144$

$x^2 = 169$

$x = \sqrt{169}$

$x = 13$

b)

$y^2 + 15^2 = 17^2$

$y^2 + 225 = 289$

$y^2 = 289 - 225$

$y^2 = 64$

$y = \sqrt{64}$

$y = 8$

Q. No. 6:

i)

- There is a triangle labeled with points A, B, and C.
- Inside the triangle, there is a smaller triangle with points D, E, and F.
- The angles are marked as follows:
  - Angle FED is marked as 33°.
  - Angle FDE is marked as 33°.
  - Angle EDB is marked as 102°.

ii)

Using the Pythagorean theorem, we have:

$x^2 = 3.2^2 + 2.4^2$

$x^2 = 10.24 + 5.76$

$x^2 = 16$

$x = \sqrt{16}$

$x = 4$

So, the length of the ladder is 4 meters.

iii) a)

In this case, the diameter of the circle is twice the radius, so it is $2 \times 5 = 10$ cm. Therefore, the diagonal of the square is also 10 cm.

$s \times \sqrt{2} = 10$

$s = \frac{10}{\sqrt{2}}$

$s = \frac{10 \times \sqrt{2}}{2}$

$s = 5 \times \sqrt{2}$

So, the length of each side of the square is $5 \times \sqrt{2}$ cm.

b)

$\text{Area of sector} = \frac{90}{360} \times \pi \times 5^2$

$\text{Area of sector} = \frac{1}{4} \times 25\pi$

$\text{Area of sector} = \frac{25}{4}\pi$

$\text{Area of triangle} = \frac{1}{2} \times 5 \times 5$

$\text{Area of triangle} = \frac{25}{2}$

$\text{Area of segment} = \frac{25}{4}\pi - \frac{25}{2}$

$\text{Area of segment} = \frac{25}{4}\pi - \frac{50}{4}$

$\text{Area of segment} = \frac{25\pi - 50}{4}$

$\text{Area of segment} = \frac{25(\pi - 2)}{4}$

So, the area of each segment is $\frac{25(\pi-2)}{4}$ square cm.

Q. No. 7:

i) a)

$$l = r \times \theta$$

Given that r = 5 cm and l = 7.5 cm, we can rearrange the formula to solve for θ:

$$\theta = \frac{l}{r}$$

$$\theta = \frac{7.5}{5}$$

$$\theta = 1.5 \text{ radians}$$

b)

$$C = 2\pi r$$

Given that r = 2 cm, the circumference is:

$$C = 2\pi \times 2$$

$$C = 4\pi \text{ cm}$$

Since the area of the sector is A = 2 cm², we can find θ using the formula for the area of a sector of a circle:

$$A = \frac{\theta}{2\pi} \times \pi r^2$$

$$2 = \frac{\theta}{2\pi} \times \pi \times 2^2$$

$$2 = \frac{\theta}{2} \times 4$$

$$2 = 2\theta$$

$$\theta = 1 \text{ radian}$$

c)

$\theta = 55^\circ \times \frac{\pi}{180}$

$\theta \approx 0.96 \text{ radians}$

Now, we can use the formula for the length of an arc of a circle to find r:

$l = r \times \theta$

Given that θ ≈ 0.96 radians and l = 6 cm, we can rearrange the formula to solve for r:

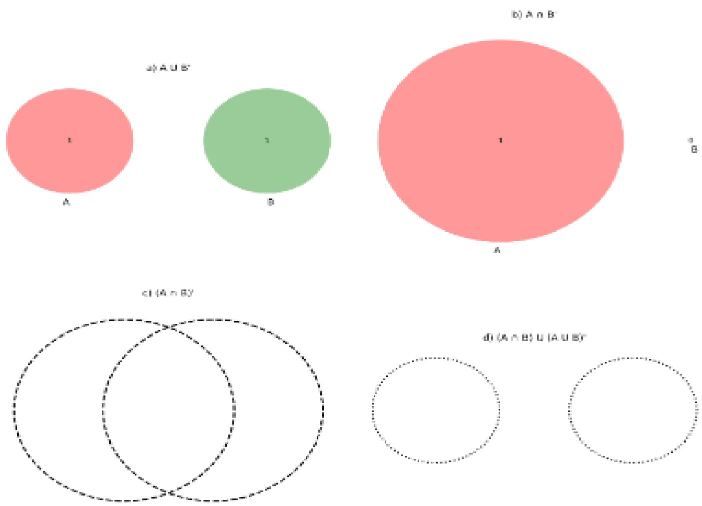
$r = \frac{l}{\theta}$

$r = \frac{6}{0.96}$

$r \approx 6.25 \text{ cm}$

So, when θ = 55° and l = 6 cm, r ≈ 6.25 cm.

ii)



Q. No. 8:

i) For a frequency of 2: | |

For a frequency of 4: | | | |

For a frequency of 8: | | | | | | | |

For a frequency of 9: | | | | | | | | |

ii) a) There are 3 ways to get a total of 10: (4,6), (5,5), (6,4).

the probability is $\frac{3}{36} = \frac{1}{12}$.

b) There is only 1 way to get a total of 12: (6,6).

the probability is $\frac{1}{36}$.

c) The outcomes are (1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (3,1), (3,2), and (4,1).

the probability is $\frac{10}{36} = \frac{5}{18}$.

d) There are 6 ways to get the same number: (1,1), (2,2), (3,3), (4,4), (5,5), (6,6).

the probability is $\frac{6}{36} = \frac{1}{6}$.

e) The outcomes are (4,6), (5,5), (5,6), (6,4), (6,5), and (6,6).

the probability is $\frac{6}{36} = \frac{1}{6}$.